2B1N20072 (Pages : 2) Reg. No:..............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester B.Sc Mathematics Degree Examination, November 2020

BMT1B01– Basic Logic and Calculus – I

(2020 Admission onwards)

Time: 2.5 hours Max. Marks : 80

Section A

A maximum of 25 marks can be earned from this section

Each question carries 2 marks

1. Construct a truth table for the proposition $(p \vee q) \vee (\sim q)$.
2. Write the converse, inverse and contrapositive of the implication ' If it is raining, then it is cold'.
3. Give a counter example to disprove the statement ' The square of every real number is positive'.
4. Find the domain of the function $f(x) = \frac{2x+\sqrt{x+2}}{3x-1}$.
5. Find $\lim_{x\to 0} \frac{1-\sqrt{x+1}}{x}$.
6. Find the equation of tangent line to the graph of the function $f(x) = 3x^2$ at the point (1,3).
7. Find the linearization of $f(x) = 2x^3$ at $x = 1$.
8. Find extrema of the function $f(x) = x^2 + 1$.
9. State Rolle's theorem.
10. Find the interval in which $f(x) = x^2 + 6x + 3$ is increasing.
11. Determine where the graph of the function $f(x) = x^3 - 8x$, is concave upward and where it is concave downward.
12. Evaluate $\sum_{k=1}^{3} \frac{k-2}{k}$.
13. Find the indefinite integral $\int (2x + sec^2 x)\, dx$.
14. Find the derivative of the function $f(x) = \int_1^x \sqrt{1+x^2}\, dx$.
15. Find the average value of $f(x) = 2x^2 - 3x$ in $[0, 2]$.

Section B

A maximum of 35 marks can be earned from this section
Each question carries 5 marks

16. State and prove De Morgan's laws of logic.

17. Prove by contradiction : $\sqrt{2}$ is an irrational number.

18. Let $f(x) = \begin{cases} 0, if\ x\ is\ rational \\ 1, if\ x\ is\ irrational \end{cases}$. Prove that $\lim_{x \to 0} f(x)$ does not exist.

19. State intermediate value theorem for continuous functions and use it to prove that there exists at least one root of the equation $x^5 + 2x - 7 = 0$, in the interval (1, 2).

20. Find all asymptotes of the graph of the function $f(x) = \frac{x^2-2}{x^2-1}$.

21. Determine the constants a and b such that the curve $y = x^3 + a x^2 + b x$, has an inflexion at the point (1, –2).

22. Find f by solving the initial value problem $f'(x) = x^2 + \sin 2x$, $f(0) = 1$.

23. A car moves along a straight road with velocity function $v(t) = t^2 + 2t - 4$, $0 \le t \le 8$, where v(t) is measured in feet per second. Find the displacement of the car between t = 1 and t =3.

Section C

Answer any two questions
Each question carries 10 marks

24. (a) Define a tautology and determine whether $(p \rightarrow \sim q) \leftrightarrow (q \rightarrow \sim p)$ is a tautology.

    (b) Using the laws of logic, simplify the expression $(p \wedge \sim q) \vee q \vee (\sim p \wedge q)$

25. (a) Let $f(x) = \begin{cases} 2ax + b, if\ x < 2 \\ 3\ , \quad if\ x = 2 \\ ax - b, if\ x > 2 \end{cases}$

    Find the values of a and b which will make the function continuous on $(-\infty, \infty)$.

    (b) Find the numbers at which the function $f(x) = \frac{\sqrt{x}}{\sin x}$ is continuous.

26. (a) Sketch the graph of the function $f(x) = \frac{x^2}{x^2-1}$.

    (b) Show that $f(x) = x^3 + 2x$ has no local maximum or minimum values.

27. (a) State and prove mean value theorem for integrals.

    (b) Find the area of the region under the graph of $f(x) = 9 - x^2$ bounded by X-axis.

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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester B.Sc.Degree Examination, November 2020

BMT1C01– Mathematics – I

(2020 Admission onwards)

Time: 2 hours Max. Marks : 60

Section A

A maximum of 20 marks can be earned from this section

Answer all question. Each question carries 2 marks

1. Find the tangent line to the parabola $y = x^2 - 3x + 1$ and sketch.
2. A bus travels $2x^2$ meters in x seconds. Find $\Delta x, \Delta y$ and the average velocity during the time interval Δx for $x_0 = 3, x = 4$.
3. Let $A(x)$ be the area of a square of side length x. Show that $A'(x)$ is half the perimeter of the square.
4. Find $\lim_{x\to 2} \frac{-3x}{x^2-4x+4}$
5. Find $\frac{d}{dx}\left(\frac{4}{(x^2-1)(x+1)^2}\right)$
6. If $A = x^2$ and $\frac{dx}{dt} = 3$, find $\frac{dA}{dt}$ $at\ x = 10$.
7. Show that the parametric equations $x = at + b$ and $y = ct + d$ describe a straight line if a and c are not both zero. What is its slope?
8. Find a function which is continuous on the whole real line and differentiable at all points except at 1, 2 and 3.
9. Prove that $\int f(x)f'(x)dx = \frac{1}{2}[f(x)]^2 + c$ for any function f.
10. Show that $\sum_{i=1}^{n}[i^3 - (i-1)^3] = n^3$.
11. Show that $-3 < \int_{-1}^{2}(t^3 - 4)dt < 4$.
12. Let $F(x) = \int_2^x \frac{1}{1+s^2+s^3} ds$. Find $F'(3)$.

(Maximum Marks 20)

Section B

A maximum of 30 marks can be earned from this section

Answer all question. Each question carries 5 marks

13. Use linear approximation to calculate $\frac{1}{(2.01)^2+(2.01)^3}$.

14. Find $\frac{d^2}{dx^2}(\frac{x}{\sqrt{1+x^2}})$.

15. Find the equation of the line tangent to the parametric curves $x = (1 + t^3)^4 + t^2$, $y = t^5 + t^2 + 2$ at $t = 1$.

16. Find maximum and minimum points and values for $y = x^4 - x^2$ on $[-1, \infty)$.

17. Find the intervals on which $f(x) = x^3 - 3x^2 + 2x$ is increasing and decreasing.

18. Find the area of the region between the graphs of $y = x^2$ and $y = x + 3$ on $[-1,1]$.

19. Find the volume of the solid obtained by revolving the region under the graphs of $sinx$ and x on $[0, \frac{\pi}{2}]$ about the x- axis.

(Maximum Marks 30)

Section C

(Answer any One Question. Each carries *ten* Marks)

20. a) Evaluate $\lim_{x\to 0}(\frac{1}{xsinx} - \frac{1}{x^2})$.

    b) Suppose that f is continuous on $[0,3]$, that f has no roots on the interval, and that $f(0) = 1$. Prove that $f(x) > 0$ for all x in $[0,3]$.

21. a) Find the area under the graph of $f(x) = x$ for $1 \le x \le 2$ using upper and lower sums.

    b) Find $\overline{cos\theta}_{[\pi,\pi+\theta]}$ as a function of θ and evaluate the limit as $\theta \to 0$.

(1 x 10 = 10 Marks)