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B5N18300 (Pages : 2) Reg. No:..............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fifth Semester B.Sc Mathematics Degree Examination, November 2018

MAT5B05 – Vector Calculus

(2015 Admission onwards)

Max. Time: 3 hours Max. Marks: 120

Section A

Answer all twelve questions (1 – 12)

Each question carries 1 Mark.

1. Define limit of a function of two variables.
2. Define critical point of a function of two variables.
3. Find the limit $\lim_{(x,y)\to(0,0)} \frac{4x^2-4y^2}{x-y}$
4. Find the rate of change of the function $f(x,y) = x^2 + y^2 + 2x + 4$ in the direction of the vector $\hat{\imath}$.
5. Define exact differential form.
6. State the tangential form of Green's Theorem in the plane.
7. What is the direction of maximum rate of change of a function $f(x,y)$ at a point P?
8. Define flux across a plane curve.
9. If C is the unit circle with centre at the origin, then what is the value of the integral $\int_C xdy - ydx$
10. Find the jacobian of the transformation $x = r\cos\theta, y = r\sin\theta$.
11. What are the level curves of the function $f(x,y) = x + y$
12. Define potential function for a vector field.

(12 x 1 = 12 Marks)

Part B

Answer any Ten from the following fourteen questions (13 – 26).

Each question carries 4 Marks

13. Find the linearization of the function $f(x,y) = x^3y^4$ at (1,1)
14. Find local extreme values of $f(x,y) = x^2 + y^2$.
15. Find $\lim_{(x,y)\to(0,0)} \frac{y^2}{x^2+y^2}$.
16. Find the line integral of $f(x,y,z) = x + y$ over the line segment $x = t, y = 1 - t, z = 0$ from (0,1,0) to (1,0,0).
17. Find a linearization of $f(x,y) = xy + 2yz - 3xz$ at the point (1,1,0).
18. Find the area of the ellipse $\vec{r}(t) = a\cos t\,\hat{\imath} + b\sin t\,\hat{\jmath}, 0 \le t \le 2\pi$.
19. Evaluate $\int_0^3\int_0^2(4-y^2)dydx$.
20. Find the equivalent polar form of the Cartesian integral $\int_0^2\int_0^x ydydx$.
21. Convert the integral $\int_{-1}^1\int_0^{\sqrt{1-y^2}}\int_0^x(x^2+y^2)dzdxdy$ to an equivalent integral in cylindrical coordinates.

22. Find the work done by $\vec{F}(x,y) = (y - x^2)\,\hat{\imath} + (z - y^2)\hat{\jmath} + (x - z^2)\hat{k}$ over the curv
$\vec{r}(t) = t\,\hat{\imath} + t^2\hat{\jmath} + t^3\hat{k}, 0 \le t \le 1$ from (0,0,0) to (1,1,1).
23. Find the counter clockwise circulation for the field $\vec{F}(x,y) = (x - y)\,\hat{\imath} + (y - x)\hat{\jmath}$ a the boundary of the curve bounded by the lines $x = 0, y = 0, x = 1$ and $y = 1$ by using Green's theorem
24. Show that Curl grad $f = 0$
25. Find the derivative of the function $f(x,y) = 2xy - 3y^2$ at $P_0(5,5)$ in the directior $\vec{A} = 4\hat{\imath} + 3\hat{\jmath}$.
26. Find the quadratic approximation of the function $f(x,y) = y\,sinx$ near the origin.

(10 x 4 =40

Part C

Answer any Six from the following nine questions (27 – 35).
Each question carries 7 Marks.

27. Find the local maxima and local minima of the function
$f(x,y) = x^2 + xy + y^2 + 3x - 3y + 4$.
28. Using chain rule express $\frac{\partial w}{\partial u}$ and $\frac{\partial w}{\partial v}$ in terms of u and v, if $w = \ln(x^2 + y^2 + z^2)$, $x = ue^v \sin u,\ y = ue^v \cos u,\ z = ue^v$. Also evaluate $\frac{\partial w}{\partial u}$ and $\frac{\partial w}{\partial v}$ at the point $(u,v) = (-2,0)$.
29. Find the parametric equations for the line tangent to the curve of intersection of the surfaces $x^2 + y^2 = 2, x + z = 4$ at the point (1,1,3)
30. Evaluate the line integral $\int_{(0,0,0)}^{(1,2,3)} 2xydx + (x^2 - z^2)\,dy - 2yzdz$.
31. Find the work done by $\vec{F} = (4x - 2y)\hat{\imath} + (2x - 4y)\hat{\jmath}$ in moving a particle once clockwise around the circle $(x - 2)^2 + (y - 2)^2 + (z - 2)^2 = 4$.
32. Find the area of the cap cut from the hemisphere $x^2 + y^2 + z^2 = 2, z \ge 0$ by the cylinder $x^2 + y^2 = 1$.
33. Find the flux of $\vec{F} = xy\hat{\imath} + yz\hat{\jmath} + xz\hat{k}$ outward through the surface of the cube cut the first octant by the planes $x = 1, y = 1$ and $z = 1$.
34. Find the volume of the region D enclosed by the surfaces $z = x^2 + 3y^2$ and $z = 8 - x^2 - y^2$.
35. Evaluate $\int_0^1 \int_0^{1-x} \sqrt{(x + y)}(y - 2x)^2\, dydx$.

(6 x 7 = 42

Part D

Answer any two from the following three questions (36 - 38).
Each question carries 13 Marks.

36. The plane $x + y + z = 1$ cuts the cylinder $x^2 + y^2 = 1$ in an ellipse. Find the po the ellipse that lie closest to and farthest from origin.
37. Verify the divergence theorem for the field $\vec{F} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ over the sphere $x^2 + y^2 + z^2 = 9$.
38. Evaluate the circulation of the field $\vec{F} = x^2y^3\hat{\imath} + \hat{\jmath} + z\hat{k}$ around the curve C, wher the intersection of the cylinder $x^2 + y^2 = 4$ and the hemisphere $x^2 + y^2 + z^2 = 1$ $z \ge 0$, by using Stoke's theorem.

(2 x 13 = 26

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35N18301 (Pages : 2) Reg. No:............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fifth Semester B.Sc Mathematics Degree Examination, November 2018

MAT5B06 – Abstract Algebra

(2015 Admission onwards)

Max. Time: 3 hours Max. Marks: 120

Section A

Answer all the twelve questions

Each question carries 1 mark.

1. Define binary operation on a set. Give one example.
2. What is the inverse of $A = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$ in the group $\mathbb{M}_2(\mathbb{R})$ under matrix addition.
3. List all subgroups of $(\mathbb{Z}_4, +_4)$.
4. Define cyclic group.
5. Give an example of an infinite non-abelian group.
6. S_3 has exactly ______ subgroups of order 2.
7. Define odd permutation.
8. The index $(2\mathbb{Z}: 4\mathbb{Z})$ is ______.
9. Define normal subgroup H of a group G.
10. Give an example of a ring without unity.
11. Characteristic of the ring $(\mathbb{Z}, +, .)$ is _______.
12. Give an example of a finite field.

(12×1=12 marks)

Section B

Answer any ten out of fourteen questions.

Each question carries 4 marks.

13. Is $\mathbb{M}_2(\mathbb{R})$, the collection of all 2×2 real matrices under multiplication a group? Justify your answer.
14. Show that the groups $(\mathbb{Z}, +)$and $(2\mathbb{Z}, +)$ are isomorphic.
15. Show that subgroup of an abelian group is abelian. Give an example of a non-abelian group in which all proper subgroups are abelian.
16. Let G be a group and suppose $a * b * c = e$, for $a, b, c \in G$. Show that $b * c * a = e$
17. Find the $gcd(360,420)$
18. Define permutation of a set. Is $f: \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^2$ a permutation ? Justify your claim.
19. Show that $\mathbb{Z}_p$ has no proper nontrivial subgroups if p is a prime number.
20. Find the orbits of the permutation $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 4 & 1 & 5 & 2 & 6 & 8 & 7 \end{pmatrix}$
21. Find the order of the permutation $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 1 & 5 & 3 & 4 & 6 & 8 & 7 \end{pmatrix}$ in S_8.
22. Show that the group homomorphism $\varphi: G \to G'$is one-to-one map iff $Ker(\varphi) = \{e\}$.
23. Find all cosets of the subgroup $< 4 >$ of $\mathbb{Z}_{12}$
24. Define units in a ring R. Describe all units in the ring $\mathbb{Z} \times \mathbb{Z}_2$
25. Solve the equation $3x = 2$ in the field $\mathbb{Z}_{23}$.
26. Define field of quotient of an integral domain. Give one example.

(10×4=40 marks)

Section C

Answer any six out of nine questions

Each question carries 7 Marks

27. Let $(a * b)^2 = a^2 * b^2$ for a and b in a group G. Show that $a * b = b * a$
28. Prove that a cyclic group with only one generator can have at most two elements.
29. Let H and K are subgroups of a group G. Show that $H \cap K$ is also a subgroup of G.
30. Let $G \cong G'$ and G is abelian. Show that G' is also abelian.
31. Show that every group is isomorphic to a group of permutations.
32. Let H is a subgroup of G and let $x \in G$. Show that $xHx^{-1} = \{ xhx^{-1} : h \in H \}$ is a subgroup of G.
33. In a ring $(R, +, .)$. Prove the following :
    a) $a \cdot 0 = 0 = 0 \cdot a$ for all $a \in R$
    b) $a \cdot (-b) = (-a) \cdot b = -(a \cdot b)$ for all $a, b \in R$
34. Define zero divisors in a ring $(R, +, .)$. Find all zero divisors in the ring $(\mathbb{Z}_6, +_6, \times_6)$
35. Show that every finite integral domain is a field.

(6×7=42 marks)

Section D

Answer any two out of three questions

Each question carries 13 Marks

36. Let G be cyclic group with generator a. Prove the following

    a) If G has infinite order , then $G \cong \mathbb{Z}$

    b) If G has finite order n, then $G \cong \mathbb{Z}_n$

37. Show that subgroup of a cyclic group is always cyclic. Describe all cyclic subgroups o the group $(\mathbb{Z}_{12}, +_{12})$
38. Show that every field is an integral domain. Is $\mathbb{Q}(\sqrt{2}) = \{a + b\sqrt{2} : a, b \in \mathbb{Q}\}$ a field Justify your answer.

(2×13=26 marks

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B5N18302 (Pages : 2) Reg. No:.............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fifth Semester B.Sc Mathematics Degree Examination, November 2018

MAT5B07 – Basic Mathematical Analysis

(2015 Admission onwards)

Max. Time: 3 hours Max. Marks: 120

Section A

Answer all the twelve questions

Each question carries 1 mark

1. Define supremum of a set.
2. The symmetric difference $A \oplus B$ = ----------------------.
3. Give an example of a bounded sequence which is not Cauchy.
4. If (x_n) is a bounded decreasing sequence, then $\lim (x_n)$ = -----------.
5. State Archimedean property of real numbers.
6. What is the set of all cluster points of the set (3, 4).
7. If $0 < x < 1$, then $\lim (x^n)$ = ---------------.
8. Define a contractive sequence.
9. Find Arg (z), if $z = -1 + i$
10. Represent $1 - i$ in polar form.
11. Identify the region $|z - 1| = 4$.
12. Find the imaginary part of $\frac{5-2i}{4+i}$. **(12x1 = 12 marks)**

Section B

Answer any ten questions out of fourteen questions

Each question carries 4 marks

13. Show that the set N x N is denumerable.
14. If $a \geq 0$ and $b \geq 0$, prove that $a < b$ if and only if $a^2 < b^2$.
15. Show that any nonempty finite subset of R contains its supremum.
16. Determine the set of all x satisfying $|x + 1| + |x| = 7$.
17. State and prove arithmetic-geometric inequality.
18. Prove that $5^n - 4n - 1$ is divisible by 16 for all $n \in N$.
19. Using the definition of limit, prove that $\lim \frac{3n+1}{5n-2} = \frac{3}{5}$.
20. Show that subsequences of a convergent sequence are convergent.
21. Prove by an example that sum of two divergent sequences need not be divergent.
22. Define ultimate property of a sequence. Give example of an ultimately monotone sequence.
23. Is the union of any collection of closed sets is closed? Justify your answer.
24. Discuss the convergence of the sequence $(n!/n^n)$
25. Prove that $\sqrt{2}\ |z| \geq |Re\ z| + |Im\ z|$.
26. Find the two square roots of $\sqrt{3} + i$. **(10 x 4 = 40 marks)**

Section C

Answer any six questions out of nine questions
Each question carries 7 marks

27. State and prove nested interval property of real numbers.

28. Show that the set of real numbers R is not countable.

29. State and prove density theorem of real numbers.

30. Test the convergence of the sequences (i) $\left(\frac{n}{3n}\right)$ (ii) $\left(\frac{\sqrt{n^2+2}}{\sqrt{n}}\right)$

31. State and prove squeeze theorem and use it to prove that $\left(\frac{\sin n}{n^2}\right)$ is convergent.

32. Let $x_1 = 1$ and $x_{n+1} = \sqrt{2+x_n}$ for $n \in N$. Show that x_n converges and find its limit.

33. Define Cantor set. Show that Cantor set contains uncountable number of points.

34. Find all 7^{th} roots of unity and exhibit them geometrically.

35. Prove that $|z_1 + z_2| \leq |z_1| + |z_2|$.

(6 x 7= 42

Section D

Answer any two questions out of three questions
Each question carries 13 marks

36. (a) State and prove the Cauchy's convergence Criterion for sequences.
(b) Apply Cauchy's convergence Criterion to show that the sequence (a_n), where $a_n = 1 + 1/2 + 1/3 + \ldots\ldots\ldots + 1/n$ is not convergent.

37. Prove that a subset of R is open if and only if it is the union of countably many disjoir open intervals in R.

38. Sketch the following regions. In each case state whether they are open or closed, whic them are connected and which of them are bounded.

(i) $|z+1| + |z-1| < 4$ (ii) $\text{Re } z^2 \leq 2$ (iii) $|3z+2| > 4$ (iv) $|z-i| > |z-1|$

(2 x 13 = 2

5N18303 (Pages : 2) Reg. No:..............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fifth Semester B.Sc Mathematics Degree Examination, November 2018

MAT5B08 – Differential Equations

(2015 Admission onwards)

Max. Time: 3 hours Max. Marks: 120

Section A

Answer all the twelve questions

Each question carries 1 mark

1. State whether the differential equation $(1+y^2)\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = e^x$ is linear or nonlinear in the variable y.
2. Find the integrating factor of the differential equation $x\frac{dy}{dx} + y = \tan x$
3. Write the standard form of a second order linear non-homogenous differential equation.
4. Define the wroskian of the two functions f(t) and g(t).
5. What is the period of the function f(x) = sin(2x) ?
6. Write the one dimensional heat conduction equation.
7. What is the Laplace transformation of the function f(t)=5t-3?
8. Define the convolution of two functions f(t) and g(t).
9. True or False: If the function f(x) is even , then its reciprocal function is also even.
10. Define impulse function.
11. Check whether the function $f(x) = x^2 + \cos(x)$ is even or odd.
12. Find the eigen values of the matrix $A = \begin{bmatrix} 4 & 0 \\ 0 & 3 \end{bmatrix}$

(12 x 1 = 12 Marks)

Section B

Answer any TEN questions

Each question carries 4 marks

13. Solve the differential equation $\frac{dy}{dx} - y = 0$.
14. Check the exactness of the differential equation $(y\cos x + 2xe^y) + (\sin x + x^2 e^y - 1)\frac{dy}{dx} = 0$.
15. State some differences between linear and non-linear differential equations.
16. Solve the initial value problem $y'' - y = 0$, $y(0) = 2$, $y'(0) = -1$
17. Find the wroskian of the functions e^x and xe^x.
18. Use the method of reduction of order to find a second a solution $y_2(x)$ of the differential equation $x^2 y'' - xy' + y = 0$ if one solution is given by $y_1(x) = x$.
19. Find the Laplace transformation of the function $f(t) = sin^2$ (t).
20. Define the unit step function u_c (t) and find its Laplace transform.
21. Find the convolution of the functions e^t and e^{-t} .
22. Find the inverse Laplace transformation of $\frac{s-4}{s^2-4}$.
23. Find the eigen values of the matrix $A = \begin{bmatrix} 5 & -1 \\ 3 & 1 \end{bmatrix}$.
24. Write the Euler-Fourier formulas to find the Fourier coefficients a_0 , a_n and b_n in the Fourier series expansion of a function having period 2L.
25. Show that the product of two odd functions is an even function.
26. Show that Laplace transformation is a linear operator.

(10 x 4 = 40 Marks)

Section C

Answer any SIX questions

Each question carries SEVEN marks.

27. Solve by the method of variation of parameters $y''-4y'+4y = \frac{e^{2x}}{x}$.
28. Solve the non-homogenous equation $y''+4y = 8x^2$ by the method of undetermin
    coefficients.
29. Use Euler's method with h=0.1 to find approximate values of the solution of th
    differential equation $\frac{dy}{dt} = \frac{3t^2}{3y^2-4}$ at t=1.2 , 1.4 , 1.6 and 1.8
30. Show that the differential equation $(2xy + y - \tan y)dx + (x^2 - x\tan^2 y + sec^2$
    is exact and hence solve it.
31. Solve the initial value problem $y''-2y'+10y=0$, y(0)=4 , $y'(0) = 1$.
32. Show that the convolution of two functions is commutative.
33. Solve the integral equation $y(t) = t+\int_0^t y(u)\sin(t-u)\,du$
34. Solve by the method of separation of variables $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$.
35. Find the Fourier Series for f(x)=|x| in $[-\pi , \pi]$.

(6 x 7 = 42

Section D

Answer any TWO questions

Each question carries 13 marks.

36. Find the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{bmatrix}$.
37. Using Laplace transformation solve $y'' + y = 3\cos(2t)$, y(0) = 0 , $y'(0) = 0$.
38. Find the Fourier series expansion of $f(x) = \begin{Bmatrix} -k, -\pi < x < 0 \\ k,\ 0 < x < \pi \end{Bmatrix}$, $f(x+2\pi) = f($

    Hence deduce that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots\ldots = \frac{\pi}{4}$.

(2 x 13 = 2

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B5N18304 (Pages : 2) Reg. No:..............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fifth Semester B.Sc Mathematics Open Course Degree Examination, November 2018

MAT5D03 – Mathematics for Social Science

(2015 Admission onwards)

Max. Time: 2 hours Max. Marks: 40

Section A

Answer all the six questions

Each question carries 1 mark

1. Complete the square of the expression $x^2 - 5x$.
2. Find the y – intercept and x – intercept for the equation $y = -4x + 8$.
3. Given $f(x) = x^2 + 6x + 8$, find $f(a)$ and $f(a+3)$.
4. Define an Inflection point.
5. Solve $\log_5 x = 3$.
6. Evaluate $\int_4^{36} \frac{dx}{\sqrt{x}}$

(6 x 1 = 6 marks)

Section B

Answer any five out of seven questions

Each question carries 2 marks.

7. Find the first- order partial derivative for $f(x,y) = \frac{x^2+y^2}{5x+2y}$
8. Use integration by substitution to evaluate $\int \frac{56x}{(7x^2+4)^3} dx$.
9. Find the critical value and determine whether the critical value is relative maximum or minimum for $f(x) = 2x^3 - 24x^2 + 72x - 15$.
10. Solve the system of equations $y = -2x + 10$; $y = \frac{1}{4}x + 1$ graphically.
11. Evaluate $\lim_{x\to 3} \frac{\sqrt{x}-\sqrt{3}}{x-3}$.
12. If the total cost function $C(x) = 0.5x^2 + 1.5x + 8$, find the marginal cost at $x = 4$.
13. Find the successive derivatives for the function $y = (5x-9)^3$.

(5 x 2 = 10 marks)

Section C

Answer any three *out of five questions*
Each question carries 4 *marks.*

14. Use logarithmic differentiation to find the derivative of
$g(x) = (x^4 + 7)(x^5 + 6)(x^3 + 2)$.

15. Find the vertex and axis of the parabola $y = x^2 - 8x + 19$ and then draw the parabola.

16. Find the volume V of the solid of revolution generated by revolving around the x the regions of the curve $f(x) = 5x^2$; $a = 1, b = 3$.

17. Find the cross partial derivatives of $z = e^{x^2y^3}$.

18. Find the break-even for a firm operating on monopolistic competition, given that revenue is $R = 48x - x^2$ and total cost is $T C = 6x + 120$.

(3 x 4 = 12 m

Section D

Answer any two *out of three questions*
Each question carries 6 marks.

19. Given the total revenue function R from the sale of x units, R(x) = 150x − 3x 2 ,

(a) The average revenue (AR) of sales between x = 15 and x = 20.

(b) The AR of sales for a small increase of sales starting at x = 15.

(c) The marginal cost (MR) at x = 15

20. Find the level of output at which profit π is maximized, given that the total revenue $R = 6400Q - 20Q^2$ and total cost $C = Q^3 - 5Q^2 + 400Q + 52{,}00$ assume Q > 0.

21. Find the equation for the line passing through (−2, 5) and parallel to the line ha the equation $y = 3x + 7$.

(2 x 6 = 12 m