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1B4M18196 (Pages : 3) Reg. No:.............................

Name: ....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fourth Semester B.Sc Degree Examination, March 2018

MAT4B04 – Theory of Equations , Matrices & Vector Calculus

(2016 Admission onwards)

Max. Time: 3 hours Max. Marks: 80

PART – A

Answer *all* questions. Each question carries *one* mark

1. If the roots of $6x^3 - 11x^2 - 3x + 2 = 0$ are in arithmetic progression, then find an equation whose roots are in harmonic progression.
2. Find the real root of $ax^3 + bx^2 + cx + d = 0$ if $2 + 3i$ is a root.
3. If α, β, γ are the roots of $x^3 + px^2 + qx + r = 0$, find $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$.
4. If α is a multiple root of $f(x) = 0$, then it must be a root of the equation ----------
5. Find the rank of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.
6. Find the row reduced Echelon form of the Matrix $\begin{bmatrix} 1 & -3 & 17 \\ 3 & 16 & -9 \end{bmatrix}$.
7. Find the Characteristic roots of $\begin{bmatrix} a & h & g \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$.
8. Write the normal form of $\begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.
9. If $\boldsymbol{a} = -\boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k}$ find $\left|\frac{1}{3}\boldsymbol{a}\right|$.
10. Check the continuity of the vector function $\mathbf{r}(t) = (\cos t)\boldsymbol{i} + (e^t)\boldsymbol{j} + \left(\frac{1}{1-t}\right)\boldsymbol{k}$.
11. Show that the vector $\mathbf{u}(t) = (\sin t)\boldsymbol{i} + (\cos t)\boldsymbol{j} + 2\boldsymbol{k}$ is orthogonal to its derivative.
12. Find κ if $\mathbf{v}(t) = (t^2)\boldsymbol{i} + t\,\boldsymbol{j}, \quad t > 0$

($12 \times 1 = 12$ *Marks*)

PART – B

Answer *any nine* questions. Each question carries *two* marks

13. Solve the equation $4x^4 - 8x^3 + 7x^2 + 2x - 2 = 0$, given that $1 + i$ is a root.
14. Find the rational roots of $2x^3 - 3x^2 - 11x + 6 = 0$
15. If α, β, γ are the roots of $x^3 + ax^2 + bx + c = 0$, form the equation whose roots are $\alpha\beta, \beta\gamma, \gamma\alpha$
16. Solve $x^3 + 4x^2 - 12x - 27 = 0$, given that its roots are in geometric progression.
17. Solve the homogeneous system of equations $x - 2y + 3z = 0,\ 2x + 5y + 6z = 0$
18. Show that the eigen values of a Hermitian matrix are all real.

19. If λ is an eigen value of a matrix A, then show that λ^2 is an eigen value of A^2

20. Find A^2, using Cayley Hamilton theorem, if $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$

21. Find the parametric equation of the line passing through the point $P = (-2, 0, 4)$ and parallel to the vector $2\boldsymbol{i} + 4\boldsymbol{j} - 2\boldsymbol{k}$

22. Find the distance of the point $(2,1,3)$ from the line $x = 2 + 2t, y = 1 + 6t, z = 3$

23. Find the spherical coordinate equation for the cone $z = \sqrt{x^2 + y^2}$

24. Find the length of the curve $\mathbf{r}(t) = (2+t)\boldsymbol{i} + (t+1)\boldsymbol{j} + t\boldsymbol{k}$ from $t = 0$ to $t = 3$

($9 \times 2 = 18$ *Marks*)

PART – C

Answer *any six* questions. Each question carries *five* marks

25. In an equation with rational coefficients, Show that the roots which are quadratic surds occur in conjugate pairs

26. Solve the equation $4x^4 - 4x^3 - 25x^2 + x + 6 = 0$, given that the difference between two of its roots is unity.

27. Find the rank of $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & 4 \\ 4 & 3 & 2 & 1 \end{bmatrix}$ by reducing in to its normal form

28. If A is a square matrix of order n prove that the sum of all the Eigen values of A is the trace of A

29. Show that the system of equations $x + y + z = a,\ 3x + 4y + 5z = b, 2x + 3y + 4z = c$ has no solution if $a = b = c = 1$

30. Find the eigen values and the eigen vector corresponding to any one of the eigen values of $\begin{bmatrix} 5 & 1 & -1 \\ 1 & 3 & -1 \\ -1 & -1 & 3 \end{bmatrix}$

31. Find the angle between the velocity and acceleration vectors at $t = 0$, if the position vector of the particle at time t is $\mathbf{r}(t) = (3t+1)\boldsymbol{i} + (\sqrt{3}t)\boldsymbol{j} + t^2\boldsymbol{k}$

32. Solve the initial value problem $\frac{dr}{dt} = (t^3 + 4t)\boldsymbol{i} + t\boldsymbol{j} + 2t^2\boldsymbol{k}, \mathbf{r}(0) = \boldsymbol{i} + \boldsymbol{j}$

33. Show that the curvature of a circle of radius a is $\frac{1}{a}$

(× 5 = 30 Marks)

PART – D

Answer *any two* questions. Each question carries *ten* marks

34. (i) Show that the equation $x^7 + 3x^5 + 5x - 9 = 0$ has exactly six imaginary roots

(ii) If α, β, γ are the roots of the equation $x^3 + qx + r = 0$, find the equation whose roots are $\beta\gamma + \alpha, \gamma\beta + \alpha, \alpha\beta + \gamma$

35. (i) If $A = \begin{bmatrix} 1 & -1 & -1 \\ 1 & 1 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ Find non-singular matrices P and Q such that PAQ is in the normal form

(ii) Solve the system of equations $4x + y + 2z = 0, -3x + 2y + 4z = 0, 8x - y - 2z = 0$

36. (i) Find the parametric equation of the line in which the planes $x + 2y + z = 1$ and $x - y + 2z = -8$ intersect.

(ii) Find $\mathbf{B}, \kappa$ and τ for the space curve $\mathbf{r}(t) = (\sin t)\boldsymbol{i} + (\sqrt{2}\cos t)\boldsymbol{j} + (\sin t)\boldsymbol{k}$

($\times 10 = 20$ Marks)

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B4M18197 (Pages : 2) Reg. No:.............................

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FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Fourth Semester B.Sc Degree Examination, March 2018

MAT4C04 – Mathematics

(2016 Admission onwards)

Max. Time: 3 hours Max. Marks: 80

Part A

Objective type.

Answer all the twelve questions.

1. Apply the operation $D^2 - 5D$ to the function $e^{2x} - 3$.
2. Write the general form of second order linear differential equation.
3. Show that e^{ix} is a solution of the differential equation $y'' + y = 0$.
4. Find $\mathcal{L}[e^{2+3t}]$.
5. State the shifting property of Laplace Transforms.
6. Find $\mathcal{L}^{-1}[\frac{1}{s^2+4}]$.
7. What is the smallest period of the function $f(x) = \sin x$?
8. Define an even function and give an example.
9. Write the one dimensional wave equation for the deflection of a string.
10. Write Trapezoidal rule for approximating $\int_a^b f(x)dx$.
11. Define Dirac Delta function.
12. Find the Wronskian of the functions $\sin x$ and $\cos x$.

(12 × 1=12 Marks)

Part B

Short answer type.

Answer any nine questions.

13. Solve the differential equation $y'' - 5y' - 6y = 0$.
14. Find a general solution of $x^2y'' - 5xy' + 8y = 0$.
15. Find $\mathcal{L}[\cos^2 t]$.
16. Find the inverse Laplace transform of $\frac{s}{(s+3)^2+1}$.
17. Write the Euler formulae for the Fourier coefficients of a function $f(x)$.
18. What do you meant by a piecewise continuous function? Give an example.

19. Prove that product of two odd function is even.

20. Solve the partial differential equation $u_{xx}=0$.

21. Find a second order linear differential equation having solutions e^{3x} and xe^{3x}.

22. Find a basis of solutions for the differential equation $x^2y'' - xy' + y = 0$ if $y_1 = x$ is one of the solutions.

23. Define the unit step function, $u_a(t)$ where a is a positive real number. Find the Laplace transform of $(t-\pi)u_\pi(t)$.

24. Use Simpson's rule to approximate $\int_0^1 \frac{1}{1+x^2} dx$ taking $n = 4$.

(9 × 2=18 Marks)

Part C
Short essay type.
Answer any six questions.

25. Solve the initial value problem $y'' - 4y' + 4y = 0; y(0) = 2, y'(0) = 1$.

26. Find a general solution of $y'' - 5y' + 4y = 2e^{3x}$.

27. Using the method of variation of parameters, find a general solution of $y'' + y = \sec x$.

28. Find the Laplace transform of $\frac{1-e^{2t}}{t}$.

29. Using the method of convolution, find the inverse Laplace transform of $\frac{1}{(s^2+1)^2}$.

30. Find the Fourier Sine series of the function $f(x) = x; 0 < x < 2$.

31. Find an upper bound for the error incurred in estimating $\int_0^1 2x^5 \ dx$ using Simpson's rule with 4 steps.

32. Apply improved Euler's method to find $y(1)$ if $y' = 2x, y(0) = 0$. (Take $h = 0.2$).

33. Use Picard's iteration method to find approximate solution of the initial value problem $y' = x - 2y; y(0) = 1$ with 3 steps.

(6 × 5=30 Marks)

Part D
Essay type.
Answer any two questions.

34. Using Laplace transforms solve the initial value problem $y'' + 4y' + 3y = 0; \ y(0) = 3, y'(0) = 1$.

35. Represent the function $f(x) = x$ in the interval $[-\pi, \pi]$ as a Fourier series and deduce that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots\ldots\ldots = \frac{\pi}{4}$.

36. Obtain d'Alemberts solution of wave equation.

(2 × 10=20 Marks)