B3N18198 (Pages : 2) . Reg. No:.....

Name: .....

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

Third Semester BSc Degree Examination, November 2018

BMAT3B03 - Calculus and Analytic Geometry

(2016 Admission onwards)

Max. Time: 3 hours Max. Marks: 80

PART-A

Answer *all* Questions . Each question carries *one* mark

1. Evaluate $\int_{-3}^{-2} \frac{dx}{x}$
2. Define hyperbolic sine function of x.
3. Find the derivative of $\tanh \sqrt{1+t^2}$
4. Write the n^{th} term of the sequence 0,3,8,15,24,....
5. Give an example of bounded non decreasing sequence.
6. State rearrangement theorem for absolutely convergent series.
7. Sketch the region $4x^2 + y^2 \leq 4$
8. Using discriminant test identify the conic section, $xy - y^2 - 5y + 1 = 0$
9. Write-down the standard parametrization of an ellipse.
10. Graph the polar region $-1 \leq r \leq 1, 0 \leq \theta \leq \frac{\pi}{2}$
11. Write the equation slope $\frac{dy}{dx}$ of the curve $r = f(\theta)$ at (r, θ)
12. Evaluate $\lim (3n)^{\frac{1}{n}}$

(12 x 1 = 12 marks)

PART-B

Answer any *nine* questions. Each question carries two marks

13. Find the derivative of $y = \log_{10}(3x + 1)$
14. Show that $\frac{d}{dx}(\tanh^{-1} x) = \frac{1}{1-x^2}, |x| < 1$
15. Evaluate $\lim_{x \to 0^+} x \cot x$
16. Evaluate $\int_0^{1/3} \frac{6\,dx}{\sqrt{1+9x^2}}$
17. Show that the sequence $\left(\frac{1}{n}\right)$ converges to 0.
18. Does the sequence with n-th term $a_n = \left(\frac{n-2}{n}\right)^n$ converge? If so what is the limit?
19. Find the sum of the series $\sum_{n=0}^{\infty} \frac{2^{n+1}}{5^n}$
20. Check the convergence of the series $\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n^2}\right)^n$
21. Find the foci and asymptotes of the hyperbola $\frac{x^2}{4} - \frac{y^2}{5} = 1$
22. Describe the motion a particle whose position $P(x, y)$at time t is given by,
$$x = a\cos t, y = b\sin t, 0 \leq t \leq 2\pi$$
23. Find the polar equation of a straight line L such that the foot of the perpendicular from the pole meet L at $\left(2, \frac{\pi}{3}\right)$.

24. Find the equation of the hyperbola with eccentricity $\frac{3}{2}$ and directrix, $x = 2$.

(9 x 2 = 18 marks)

PART-C

Answer any *six* questions. Each question carry *five* marks

25. Prove that $\cosh^{-1} x = \ln\left(x + \sqrt{x^2 - 1}\right), x \geq 1$
26. Find the radius and interval of convergence of the power series $\sum_{n=0}^{\infty} \frac{(2x+3)^{2n+1}}{n!}$
27. Find the partial sum and hence find the sum of the series $\sum_{n=1}^{\infty} \frac{2n+1}{n^2(n+1)^2}$
28. Using the limit comparison test, check the convergence of $\sum_{n=1}^{\infty} \frac{\sqrt{n+1}-\sqrt{n-1}}{n^2}$
29. Find the Maclaurin series of $f(x) = x^2 \sin x$.
30. Find the equation of the hyperbola centered at origin that has focus at $(\sqrt{10}, 0)$ and directrix the line $x = \sqrt{2}$.
31. Find $\frac{d^2y}{dx^2}$ if $x = t - t^2, y = t - t^3$
32. Find the area of the surface generated by revolving $x = \cos t, y = 2 + \sin t, 0 \leq t \leq 2\pi$ about X-axis.
33. Find the area of the region in the plane enclosed by the cardioid $r = 2(1 + \cos\theta)$

(6 x 5 = 30 marks)

PART-D

Answer any *two* questions. Each question carries *ten* marks

34.
   a) Graph the curve $r^2 = 4\cos\theta$
   b) Show that the point $(2, \frac{\pi}{2})$ lies on the curve $r = 2\cos 2\theta$

35.
   a) Find the tangent to the right hand hyperbola branch $x = \sec t, y = \tan t, -\frac{\pi}{2} < t < \frac{\pi}{2}$ at the point $(\sqrt{2}, 1)$, where $t = \frac{\pi}{4}$
   b) Find the length of the parametric curve, $x = \cos t, y = t + \sin t, 0 \leq t \leq \pi$

36.
   a) Solve for y, in terms of x, if $\ln y = 2t + 4$
   b) Show that $\lim_{x \to 0+}(1 + x)^{\frac{1}{x}} = e$

(2 x 10 = 20 marks)