

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE
First Semester B.Sc Degree Examination, November 2017
BMAT1C01- Mathematics
(2017 Admission onwards)

Time: 3 hours

Max. Marks : 80

Part -A

Answer all questions. Each carries one mark

1. If $A = \{a, e, i, o, u\}$ and $B = \{a, b, c, d, e\}$, then $A - B = \dots\dots\dots$
2. If $|A| = 24$, $|B| = 69$ and $|A \cup B| = 81$, then $|A \cap B| = \dots\dots\dots$
3. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ and $g: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = x^2$ and $g(x) = x+3$, then $(g \circ f)(2) = \dots\dots\dots$
4. If $(x + 1, y - 3) = (3, 5)$, then $x + 2y = \dots\dots\dots$
5. Suppose $H(t) = t^2 + 5t + 1$. Find the limit $\lim_{t \rightarrow 2} H(t)$
6. The points at which the function $f(x) = |x-2|$ is continuous is $\dots\dots\dots$
7. Find the derivative of $y = \sin(\cos 2x)$ with respect to x .
8. The maximum of the function $f(x) = x^2$ in the interval $[-2, 2]$ is attained at the points
9. Find the limit as x tends to 0 from the right $\lim_{x \rightarrow 0^+} \frac{1-x^2}{x}$
10. Find the vertical asymptotes of the graph of $f(x) = \frac{3x+1}{x^2-1}$
11. State Rolle's theorem.
12. $\lim_{x \rightarrow 0} \frac{1-\cos x}{x}$ (12 x 1= 12 marks)

PART -B

Answer any seven questions. Each carries two marks

13. Define reflexive relation on a set with example.
14. If $f(x) = ax + b$ and $g(x) = cx + d$, then evaluate $(f \circ g)(x) - (g \circ f)(x)$
15. If $A = \{1, 2, 3, 4, 5\}$ and $B = \{2, 4, 6, 8\}$. Find $(A-B) \cup (B-A)$.
16. Prove that the relation defined on the set of all straight lines defined by "is perpendicular to" is not transitive.
17. Find $\frac{dy}{dx}$ if $y = (1-x)(\cos x - 1)$ at $x = \pi$.
18. Distinguish between absolute maximum and local maximum of a function.
19. Find the linearization of the function $f(x) = x^2 + 1$ at $x = 1$.
20. Evaluate $\lim_{x \rightarrow 0} x \cot x$.
21. Discuss the concavity of the function $f(x)$. (7 x 2=14 marks)

PART -C

Answer any six questions. Each carries five marks

22. Prove that the relation R defined by $a \equiv b \pmod{5}$ an equivalence relation.
23. The deflection of the a beam is given by $y = 2x^3 - 9x^2 + 12x$. Find maximum deflection.
24. Is the function $f(x) = \frac{x^2 + x - 6}{x^2 - 4}$ continuous? If not find a continuous extension.
25. Evaluate $\lim_{x \rightarrow 0} \frac{\log(1+x) - \log x}{x^2}$.
26. State sandwich theorem . Use this to find $\lim_{x \rightarrow 0} g(x)$ if $3+x^2 \leq g(x) \leq 3\cos x$.
27. Use first principle to find the derivative of \sqrt{x} .
28. Verify Mean value theorem for $f(x) = x + \frac{1}{x}$, on $\left[\frac{1}{2}, 2\right]$
29. Find the vertical asymptotes of the curve $f(x) = \frac{3x^2 + 6x + 5}{x^2 - 3x + 2}$.

(6 x 5 = 30 marks)

PART -D

Answer any three questions. Each carries eight marks

30. Consider the relation R defined on the set $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$ by $R = \{(a, b) : a \text{ divides } b\}$
- (a) Find the domain and range of R
- (b) Is this relation an equivalence relation?
- (c) Find the inverse relation.
31. (a) Use first principle to find the derivative of $\sin x$
- (b) Find the derivative of the function $\log \sqrt{\frac{1 - \cos x}{1 + \cos x}}$
32. Consider the function $f(x) = \begin{cases} 1 & \text{if } x \leq 3 \\ ax + b & \text{if } 3 < x < 5 \\ 7 & \text{if } x \geq 5 \end{cases}$. Find a and b such that f is continuous
33. Evaluate (a) $\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1}{x}$ (b) $\lim_{x \rightarrow 0^+} (e^x + x)^{\frac{1}{x}}$
34. (a) Find the intervals in which $f(x) = -x^2 - 3x + 3$ is increasing and decreasing
- (b) Find the value of c in the mean value theorem for the function $f(x) = x + \frac{1}{x}$ in $[1/2, 2]$

(3 x 8 = 24 marks)

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Reg. No:.....

Name:

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PART A

Answer all questions. Each question carries one Mark

$A \oplus B = (A \setminus B) \cup \dots\dots\dots$

If S has 10 elements then P(S) has elements.

Give an example of a relation on $A = \{1, 2, 3\}$ which is not an anti symmetric relation.

$\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1} = \dots\dots\dots$

Cardinality of the set $\{\phi\} \cup \{1, 2\}$ is

Define a unit step function.

The function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = [x]$ is not continuous at

$\bigcap_{n=1}^{\infty} \left[\frac{-1}{n}, \frac{1}{n} \right] = \dots\dots\dots$

Define a tautology.

Write the dual of the statement: $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$.

Write the contrapositive of the statement: "If $3 + \alpha = 5$ then $\alpha = 2$ or 4 ."

What is the truth value of $\forall x P(x)$, where $P(x)$ is the statement " $x^2 > -1$, the universe of discourse consist of all integers.

(12 × 1 = 12 marks)

PART- B

Answer any seven questions.

Each question carries two marks

Write all partitions of the set $\{1, 2, 3\}$

Show that (\mathbb{R}, \leq) is a poset.

At what points does the function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = |x|$ is continuous.

Let $f, g : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = x + 5$ and $g(x) = x^2 + 4, \forall x \in \mathbb{R}$. Compute $f \circ g$ and $g \circ f$

Draw the graph of the function

$$f(x) = \begin{cases} 2x & \text{if } 0 \leq x < 1 \\ 3 & \text{if } x = 1 \\ 3 - x & \text{if } 1 < x \leq 2 \end{cases}$$

For $n = 1, 2, 3, \dots$ let $A_n = (n, \infty)$ then show that $\bigcup_n A_n = (1, \infty)$ and $\bigcap_n A_n = \phi$

Show that $[(p \rightarrow q) \wedge \neg p] \rightarrow \neg q$ is a contingency.

Using De Morgan's Law find the negation of the statement "Sachin is simple and hardworking."

State the converse and inverse of the statement: If John is a film star, then he is rich.

(7 × 2 = 14 Marks)

PART- C

Answer any six questions.

Each question carries five marks

22. Show that a function $f: X \rightarrow Y$ is invertible if and only if it is bijective.
23. Using shifting of graphs, draw the graph of the function $y = (x - 1)^3 - 1$.
24. Show that the set of integers and the set of even integers are equipotent.
25. Let $A = \{1, 2, 3\}$, $B = \{a, b, c\}$, $C = \{x, y, z\}$. Consider the following relations R from A to B and S from B to C given by $R = \{(1, a), (1, c), (2, b), (2, c), (3, b)\}$ and $S = \{(a, x), (a, y), (b, z), (c, x), (b, y)\}$. Find $R \circ S$ using relation matrix of R and S .
26. Evaluate $\lim_{x \rightarrow 2^+} \frac{x^2 - 3x + 2}{x^3 - 4x}$
27. Prove that $\neg \forall x P(x) \equiv \exists x \neg p(x)$.
28. Construct the truth table for the compound proposition $(p \leftrightarrow q) \oplus (\neg p \leftrightarrow \neg r)$.
29. Using the laws of logic simplify the compound proposition $(p \wedge \neg q) \vee q \vee (\neg p \wedge q)$.
(6 × 5 = 30 Marks)

PART- D

Answer any three questions.

Each question carries eight marks

30. On the set of integers \mathbb{Z} define the relation xRy if and only if $x \equiv y \pmod{5}$. Show that R is an equivalence relation on \mathbb{Z} . Find all equivalence classes corresponding to this relation.
31. Find the domain, inverse function and range of the function $f(x) = \frac{x^2 - x + 1}{x^2 - 3x + 2}$
32. Consider the function f defined $f(x) = \begin{cases} x^2 - 1 & -1 \leq x < 0 \\ 2x & 0 < x < 1 \\ 1 & x = 1 \\ -2x + 4 & 1 < x < 2 \\ 0 & 2 < x < 3 \end{cases}$
- a) Find $f(-1)$. b) Does $\lim_{x \rightarrow 1^+} f(x)$ exist. c) Does $\lim_{x \rightarrow 1^-} f(x)$ exist.
d) Does $\lim_{x \rightarrow 1} f(x)$ exist. e) Is f continuous at $x = 1$
33. (i) Show that the set of rational numbers Q is a denumerable.
(ii) Show that $(0, 1)$ is not denumerable.
34. Are these system specifications consistent?
'The system is in multiuser state if and only if it is operating normally'
'If the system is operating normally, then the kernal is functioning'
'The kernal is not functioning or the system is in interrupt mode'
'If the system is not in multiuser state, then it is in interrupt mode'
'The System is not in interrupt mode.'
- (3 × 8 = 24 Marks)