FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester B.Sc Degree Examination, November 2016

AS1C01 – Financial Mathematics

(2016 Admission onwards)

Max. Time: 3 hours   Max. Marks : 80

PART A

Answer all 12 questions. Each question carries 1 mark

Fill in the blanks (Questions 1-6)

1. The interest for an amount of Rs. 12000 for three years with simple interest rate 9% per annum is ----

2. ----------interest rate is used when inflation rate is taken into account

3. What is $d^{(m)}$ as m tends to ∞?

4. Process of translating cash flow into equivalent dollars at common base period is considered as-----

5. The present value of annuity immediate of amount Rs.100 paid annually for 2 years at the rate of interest 9 % is ---------

6. Usual notation to denote the increasing immediate annuity is ………

Choose the correct answer (Questions 7-12)

7. Annuity policy holders are usually called

     (a) Annuters   (b) Customers

     (c) Insurers   (d) Annuitants

8. The total interest paid over the whole transaction per unit of initial loan is called

     (a) Money rate of interest   (b) Real rate of interest

     (c) Flat rate of interest   (d) Effective rate of interest

9. The effective rate per annum of 8% per annum convertible quarterly is

     (a) 0.092   (b) 0.072

     (c) 0.062   (d) 0.0824

10. Annuity for which payments are made at the beginning of the period is

     (a) Annuity immediate   (b) Annuity certain

     (c) Perpetuity   (d) Annuity due

11. Accumulated series of deposits as future sum of money is classified as

(a) Marginal fund   (b) Sinking fund

(c) Nominal fund   (d) Annuity fund

12. If desired rate of cash return exceeded by actual rate of return, then it is

(a) Positive cash flow   (b) Negative cash flow

(c) Future cash flow   (d) Present cash flow

(12 x 1 =12 marks)

PART B
Answer any 7 questions. Each question carries 2 marks

13. Give the relations between v, d , δ and i

14. Distinguish between the terms *payment accumulates* and *payment discounted* in financial mathematics.

15. Define (i) equity and (ii) zero coupon bond

16. Suppose that a capital of Rs. 500 earns Rs. 150 of interest in 6 years. What was the interest rate if compound interest is used?

17. Define perpetuity. Give the expression for present value of perpetuity.

18. Calculate $(I\ddot{a})_{\overline{8}|}$ at 8% p.a.

19. Give the basic principles of consistency.

20. Calculate the flat rate of interest paid on a loan of Rs.48, 000 that is repaid over 25 years. The monthly payment is Rs. 278 each.

21. Calculate $4/a_{\overline{10}|}$ at 4.8% per annum convertible quarterly.

(7 x 2 =14 marks)

PART C
Answer any 6 questions. Each question carries 5 marks

22. (i) Define force of interest. The force of interest is 8%, what is the effective rate of interest?

    (ii) If Rs. 2000 invested at 8% per annum, how long will it take for the amount to become Rs.3500?

23. Calculate

    i. $a^{(12)}_{\overline{3.5}|}$, given that i=19.5618%

    ii. $a^{(4)}_{\overline{13.25}|}$ at 10.3813% p.a. effective.

24. Define nominal rate of interest. If Rs.100000 are invested for 3 years for a nominal interest rate of 6%, what will be the amount be if interest is compounded continuously?

25. How long does it take to double your capital if you put it in an account paying compound interest at a rate of 7.5%? What if the account pays simple interest?

26. A man inherited Rs. 85000 which he deposited in a bank on January 1st 2000. Interest on the amount in the bank was added at the end of each year at 7%. The man withdrew Rs.7500 on January 1st 2001, 2002 and 2003 but made no additional deposits. How much had he to his credit on January 1st 2005 when interest for the year 2004 has been added?

27. A woman borrows Rs.25000 to buy a car. She pays the loan 24 monthly installments arrears. The flat rate of interest is 9% p.a. What is his monthly payment? What is the APR on this transaction?

28. Calculate the accumulated value of Rs. 8500, assuming a force of interest of 9% p.a., after

    (i) 3 months and (ii) 4 years and 10 days.

29. Derive an expression for the present value of increasing annuity.

(6 x 5 =30 marks)

PART D

Answer any 3 questions. Each question carries 8 marks

30. The force of interest $\delta(t)$ is a function of time, and at any time t, measured in years, is given by the formula.

$$\delta(t) = \begin{cases} 0.04, & 0 < t \leq 8 \\ 0.005t, & 8 < t \leq 20 \\ 0.003t + 0.0002t^2, & 20 < t \end{cases}$$

    (a) Derive expression in terms of t for the accumulated amount at time t of an investment of 1 at time 0.

    (b) Calculate the value at time 0 of Rs.100 due at time 15.

31. Derive the expression for

    (i)   The accumulated value of an immediate annuity, at the end of n years with annuity payments payable 'p' times in a period.

    (ii)   Present value of a deferred annuity immediate.

32. A loan of Rs. 6,000 is repayable over 8 years by level quarterly installments calculated using a rate of interest of 10% per annum effective.

    a) Calculate the amount of each quarterly installment.

    b) What is the capital content of the sixth repayment?

    c) How much interest is paid in the third year?

    d) Determine when the amount of loan outstanding falls below Rs. 2,000.

33. (i) Describe the basic characteristics of mortgage loan.

    (ii) A mortgage of Rs.120,000 is repaid over 20 years by equal monthly payments. How much is every payment on the basis of an effective interest rate of 5.89% p.a.? Suppose that the rate increases by one percent point after eleven years. Compute the revised monthly payment.

34. Write short notes on the following:

    (i)   Annuities over non-integer periods.

    (ii) Various forms of annuity certain.

    (iii) Fixed interest securities.

    (iv) Consumer credit transaction.

(3 x 8 = 24 marks)

FAROOK COLLEGE (AUTONOMOUS), KOZHIKODE

First Semester B.Sc Degree Examination, November 2016

MAT1B01– Foundations of Mathematics

(2016 Admission onwards)

Max. Time: 3 hours   Max. Marks : 80

PART-A

Answer all questions. Each question carries *one* mark

1. Let A be the set of all letters in the word "YES". Then the number of subsets of the set A is -------

2. A relation R on a set A is said to be transitive if --------

3. Write the dual of $(A \cup \phi) \cup (U \cap A^C) = U$

4. If A= {1,2,3,4} , B = {3,4,5,6,7} then $A \oplus B$ = ------

5. $\log_2 8$ = ------

6. If $F(x) = (x-1)^2 + 3$, then F(2)= ---------

7. If P(x) and Q(x) are polynomials and $Q(c) \neq 0$ then $\lim_{x \to c} \frac{P(x)}{Q(x)}$ = ----------

8. A function f is continuous at an interior point x = c of its domain if --------

9. State True or False : $p \vee F = p$.

10. Write the negation of the proposition ' There is no population in Mumbai' .

11. If $f(x) = \frac{x^5 - 1}{x - 1}$,then $f(1)$ = --------

12. The graph of an even function is symmetric about --------

(12 x 1 = 12)

PART-B

Answer any *seven* questions. Each question carries *two* marks

13. If A = {0,1} , B = {1,2} and C = {0,1,2}, find A× B× C .

14. If U = {1,2,3, . . . ,9} , A = {1,2,3,4,5} and B = {2,4,6,8}, then find (A-B)^c .

15. Let A = {1,2,3,4} and let R be a relation on A given by

    R={(1,1),(2,2),(2,3),(3,2),(4,2),(4,4)}. Find RoR .

16. Let A={1,2,3,4,6} and let R be a relation on A defined by 'x divides y' .Write R a
    a set of ordered pairs. Find the inverse relation R^{-1} of R.

17. If f(x) = x+5 and g(x)= x^2-3 find the composite functions fog and gof.

18. If $f(x) = \sqrt{x}$, $L = \frac{1}{2}$, $x_0 = \frac{1}{4}$,$\varepsilon = 0.1$, find $\delta > 0$, such that

    $0 < |x - x_0| < \delta \Rightarrow |f(x) - L| < \varepsilon.$

19. For what value of 'a' is f(x) = $\begin{cases} x^2 - 1 & ,x < 3 \\ 2ax & , \ x \geq 3 \end{cases}$ continuous at every x?

20. Show that $\neg p \leftrightarrow q$ and p $\leftrightarrow$ $\neg q$ are logically equivalent.

21. State and prove De-Morgan's laws for propositions.

(7x 2 = 1⸱

PART- C

Answer any *six* questions. Each question carries *five* marks.

22. Let A,B,C,D be sets. Suppose R is a relation from A to B, S is a relation from B to C
    and T is a relation from C to D. Show that Ro(SoT) = (RoS)oT.

23. Let ·S = {1,2,3,4}. Let R be a relation on S given by R=
    {(1,1),(2,2),(1,3),(3,1),(3,3),(4,4)}. Is R an equivalence relation? If yes describe the
    equivalence classes and a system of representatives of R and quotient set S/R.

24. If P is the set of positive integers ,show that PxP is countable

25. For each $n \in Z$, let $B_n = (n, n + 1]$. Find (a) $B_4 \cup B_5$ (b)$B_6 \cup B_7$

    (c)$B_7 \cup B_9$(d)$B_4 \cap B_5$ (e) $\cup_{i=4}^{20} B_i$.

26. If $\lim_{x \to 2} \frac{f(x) - 5}{x - 2} = 3$, find $\lim_{x \to 2} f(x)$.

27. Find $\lim_{h \to 0} \frac{\sqrt{(2+h)} - \sqrt{2}}{h}$.

28. Show that the proportions $p \lor (q \land r)$ and $(p \lor q) \land (p \lor r)$ are logically equivalent using truth tables tables.

29. Express the statement "No rabbit knows Calculus" using quantifiers. Also find its negation.

$(6 \times 5 = 30)$

PART D

Answer any *three* questions. Each question carries *eight* marks

30. Prove that a function f is invertible $\Leftrightarrow$ It is one-one and onto.

31. Consider the functions f: $A \to B$ and g: $B \to C$. Prove the following

    a) If gof is one to one then f is one to one.

    b) If gof is onto then g is onto.

32. Consider the relation R= $\{(1,3),(1,4),(3,2),(3,3),(3,4)\}$ on A=$\{1,2,3,4\}$. Then
    (a) Find the matrix representation M_R of R

    (b) Find the domain and range of R

    (c) Find R^{-1}

    (d) Find the composition relation RoR.

33. Suppose $P = \{A_i\}$ is a partition of a set S. Then prove that there is an equivalence relation " $\sim$ " on S such that the quotient set S/ $\sim$ of equivalence classes is the same as the partition $P = \{A_i\}$.

34. Show that

    (a) $\neg p \lor (\neg p \land q)$ and $\neg p \land \neg q$ are logically equivalent.

    (b) $p \land q \to p \lor q$ is a Tautology.

$(3 \times 8 = 24)$

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First Semester B.Sc Degree Examination, November 2016

MAT1C01– Mathematics

(2016 Admission onwards)

Max. Time: 3 hours

Max. Marks : 80

PART – A

Answer *all* questions. Each question carries *one* mark

1. If $r = \frac{2}{\sqrt{4-\theta}}$, then $\left.\frac{dr}{d\theta}\right|_{\theta=0}$ =.............

2. Find $\lim_{x\to\infty}\left(7 - \frac{1}{x}\right)$

3. If $\lim_{x\to 0} f(x) = 3$, then find $\lim_{x\to 0}\frac{f(x)\,\sec x}{x-3}$

4. At what points is $y = \frac{2x+1}{x^2-5x+6}$ continuous?

5. Differentiate $\tan(x^3 + 3)$ with respect to x

6. Find the slope of $f(x) = x^3 - 3$ at the point $(2,5)$

7. Define the absolute maximum of a function

8. Give an asymptote of the curve $y = \frac{1}{x}$

9. Find the norm of the partition $P = \{0, 0.2, 0.3, 0.7, 1\}$

10. Evaluate $\int_{\sqrt{3}}^{1/\sqrt{3}} \sqrt{3}\,dx$

11. Find the average value of $f(x) = x^3 + 2$ on $[0,5]$

12. $\frac{d}{dx}\left(\int_0^x \sin\theta\,d\theta\right)$ =............

$(12 \times 1 = 12)$

PART - B

Answer *any seven* questions. Each question carries *two* marks

13. Evaluate $\lim\limits_{x \to -2} \left(\dfrac{-2x-4}{x^3+2x^2} \right)$

14. If $\sqrt{5-4x^2} \le f(x) \le \sqrt{5-x^2}$ on $[-1,1]$, find $\lim_{x \to 0} f(x)$.

15. Show that the function $y = |x|$, has no derivative at $x = 0$

16. Find the absolute maximum and minimum values of $f(x) = \sqrt{4-x^2}, -2 \le x \le 1$

17. Suppose f is differentiable on $[0,1]$ and that its derivative is never zero. Show that

$$f(0) \ne f(1)$$

18. Find the critical points of $f(x) = x^{1/3}(x-4)$

19. Find the Linearization of $f(x) = (1+x)^k$ at $x = 0$. Use this to evaluate $(1.0002)^{50}$

20. Evaluate $\lim_{x \to 0} \dfrac{x \sin x}{1-\cos x}$

21. Show that the function f defined by $f(x) = \begin{cases} 1 & \text{when } x \text{ is rational} \\ 0 & \text{when } x \text{ is irrational} \end{cases}$ has no

Riemann integral over $[0,1]$

$$(7 \times 2 = 14)$$

PART - C

Answer *any six* questions. Each question carries *five* marks

22. For what value of λ, is $f(x) = \begin{cases} x^2 - 1 & x < 3 \\ 2\lambda x & x \ge 3 \end{cases}$ continuous at $x = 3$

23. Find the derivative of $y = \sqrt{x}, x > 0$ using the definition of derivative. Also find the slope of the curve when $x = 4$

24. The volume of a spherical balloon changes with the radius. At what rate does the volume change with respect to the radius when $r = 20$ cm.

25. Identify the local extreme values of $f(x) = (x+1)^2, -\infty < x \le 0$. Where they are assumed?

26. Find the asymptotes of the graph of $(x) = \dfrac{x^2-4}{x-1}$.

27. Find the area between the curves $y = x^2$ and $y = -x^2 + 4x$.

28. Find the derivative of $\int_0^{\sqrt{x}} \sin t \, dt$ by

(i) differentiating the value integral and (ii) by differentiating the integral directly.

29. Using L'Hopital's rule, evaluate $\lim_{x \to \infty} x^{1/x}$

$$(6 \times 5 = 30)$$

PART – D

Answer *any three* questions. Each question carries *eight* marks

30. (i) Define Right-hand and left-hand limits of a function defined an interval.

(ii) Let $f(x) = \begin{cases} 3 - x, & x < 2 \\ 2, & x = 2 \\ \frac{x}{2}, & 2 < x < 3 \\ 3, & x = 3 \\ x + 1, & x > 3 \end{cases}$

Find $\lim_{x \to 2^+} f(x)$, $\lim_{x \to 2^-} f(x)$, $\lim_{x \to 3^+} f(x)$ and $\lim_{x \to 3^-} f(x)$

Does $\lim_{x \to 2} f(x)$ exists? Does $\lim_{x \to 3} f(x)$ exists?

31. If f and g are continuous functions, then prove that $f + g$ and fg are continuous.

32. Find y' and y'' and graph the function $y = x^{5/3} - 5x^{2/3}$. Include the coordinates of local extreme points and inflection points.

33. (i) About how accurately should we measure the side of a square to be sure of calculating the area within 2% of its true value?

(ii) Find $\lim_{x \to 0} \left(\frac{1}{\sin x} - \frac{1}{x} \right)$

34. (i) Show that the value of $\int_0^1 \sqrt{x + 8} \, dx$ lies between $2\sqrt{2}$ and 3

(ii) Find the area of the region in the first quadrant that is bounded above by $y = \sqrt{x}$ and below by the x −axis and the line $y = x - 2$

$$(3 \times 8 = 24)$$