Reliability Sampling Plans Based on Generalized Exponential Distribution from Truncated Life Tests

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Abstract

In this paper, an attempt is made for developing reliability sampling plans for truncated life tests, when the distribution of failure times of underlying test products follows a two-parameter Generalized Exponential distribution (GED). On the basis of proposed sampling plan, an experimenter can save his time and cost to reach result which is to accept the submitted lot or to reject it. For various acceptance numbers, various confidence levels and various values of the ratio of the fixed experimental time to the specified average life, the smallest sample size necessary to assure a specified average life is worked out. The operating characteristic values for the sampling plans as well as producer's risk are presented. Examples are given with the help of tables to illustrate the effectiveness of the proposed method.

Key words: Reliability Sampling Plans, Generalized Exponential distribution, Consumer and Producer's risk, Operating characteristic function, Truncated life test.

1. INTRODUCTION

In many situations, lifetime (X) is one of the quality characteristics for some products and the sampling plans employed in these cases are termed as "reliability sampling plan" or "acceptance sampling based on life test" and the test is called life test. These plans are used to take decisions on the disposition of lots based on life testing of products. In reliability sampling plans, one might be interested in testing whether the average life λ of a product, is at least as large as λ_0 (a specification) against the alternative $\lambda < \lambda_0$. When the life test shows that the

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average lifetime of products exceeds or equal to the specified one, one can consider the lot as acceptable otherwise treat the lot as unacceptable and reject it.

In some situations, for products having a high reliability, the test of product life under normal use often requires a long period of time; in this case one can use the truncated life test for saving time and money. The test can be performed without waiting until all test products fail, and then the time can be reduced significantly. The problem considered is that of finding the smallest sample size necessary to assure a certain average life when the life test is terminated at a pre assigned time T, and when the observed number of failures does not exceed a given acceptance number c. The decision criteria is to accept a lot only if the specified average life can be established with a pre-assigned high probability P^*, which provides protection to the consumer. The life test is terminated at the time when the $(c+1)^{th}$ failure is observed or at the time T whichever is earlier.

Extensive work has been carried out in this type of reliability sampling plan based on truncated life test developed by several authors. Sobel and Tischendrof (1959) for exponential distribution, Goode and Kao(1961)for Weibull distribution, Gupta and Groll (1961)for gamma distribution, Kantam and Rosaiah (1998) for half logistic distribution, Rosaiah et al (2006) for exponentiated log logistic distribution, Kantam et al (2001) and Baklizi (2003) provide the time truncated life tests for log-logistic and Pareto distributions respectively.

In reliability theory and life testing model one can reasonably use several lifetime distribution, the two parameter Generalized Exponential distribution as the generalization of the Exponential distribution is one of them. In the present work, reliability sampling plans for truncated life tests are developed when the lifetime of products follows a Generalized Exponential distribution with known shape parameter. For various acceptance numbers, various confidence levels and various values of the ratio of the fixed experimental time to the specified average life, the smallest sample size necessary to assure specified average lifetime are obtained under the specified plan

2. THE RELIABILITY TEST PLAN

The two-parameter GE distribution is a particular member of the three-parameter exponentiated Weibull distribution; introduced by Mudholkar and Srivastava (1993). It has been studied quite extensively by Gupta and Kundu (1999, 2001a, 2001b, 2002, 2003a, 2003b, 2004), Raqab (2002), Raqab and Ahsanullah (2001). At present GE distribution has received special attention in the probabilistic statistical literature and in various applications. It is observed that the two-parameter GE model can be used quite effectively in analyzing positive lifetime data in place of well known Weibull, gamma or log-normal model. In different studies it has been shown that for certain ranges of the parameter values, it is extremely difficult to distinguish between GE and Weibull, gamma, log-normal, generalized Rayleigh distributions.

Suppose that n products are put on test and that their lifetimes $X_1, X_2, \ldots, X_n$ follow the generalized exponential distribution with cumulative distribution function (c.d.f)

$$F\ (x;\ \alpha,\ \lambda) = (1-e^{-x/\lambda})^{\alpha} \tag{2.1}$$

One of the objectives of this experiment is to set a lower confidence limit on the average life and ones want to test whether the average lifetime of products is longer than our expectation. The decision is to accept the lot if and only if the number of observed failures at the end of the fixed time T does not exceed a given acceptance number c or to terminate the test and reject the lot if there are more than c failures occurred before time T, which shows that the true average lifetime of products is below the specified one. For such a truncated life test and the associated decision rule, one may be interested in obtaining the smallest sample size to reach at a decision. The sampling plan contains: the number of units n on test, an acceptance number c, the maximum test duration T and the ratio T/λ_0, where λ_0 is the specified average life.

First, fix the consumer risk which is the probability of accepting a bad lot (the one for which the true average life is below the specified average life λ_0) not to exceed $1 - P^*$. Thus, the probability P^* is a confidence level in the sense that the chance of rejecting a lot with $\lambda < \lambda_0$ is at least p^*. For a fixed p^*, the given sampling plan is characterized by the triplet $(n,\ c,\ T/\lambda_0)$.

In accordance with the design of the proposed sampling plans, the problem is to determine for given values of $p^*(0< p^* <1)$, λ_0 and c, the smallest positive integer 'n' such that

$$\sum_{i=0}^{c} \binom{n}{i} p^{i}\ (1-p)^{n-i} \ \leq\ 1-P^{*} \tag{2.2}$$

where

$$p = F(T, \alpha, \lambda_0) = (1 - e^{-T/\lambda_0})^{\alpha}$$

It is clear that p depends only on the ratio T/λ_0.

Let $\lambda^* = T/\lambda_0$. Following the equation (2.1) one can show that

$$\frac{d}{d\lambda^*} F(T, \lambda) \geq 0$$

This shows that F (T, λ) is a non decreasing function of λ^*. Accordingly one can obtain

$$F(T, \lambda) \leq F(T, \lambda_0) \;\Leftrightarrow\; \lambda \geq \lambda_0$$

That is, the true average life is more than the specified average and the lot is accepted as a good lot.

The smallest sample sizes satisfying the inequality (2.2) are obtained and presented in Table 1 for T/λ_0=2.75, 3.00, 3.25, 3.50, 3.75, 4.00, 4.25, 4.50, 4.75 and P^*= 0.75, 0.90, 0.95, 0.99 for α =2.

If $p = F(T, \alpha, \lambda)$ is small and n is large, the binomial probability may be approximated by Poisson probability with parameter $\lambda = np$ so that the left side of equation (2.2) can be written as

$$\sum_{i=0}^{c} \left(\frac{e^{-\lambda} \lambda^{i}}{i!}\right) \leq 1 - P^* \tag{2.3}$$

where $\lambda = n\ F(T, \alpha, \lambda_0)$. The smallest sample sizes satisfying equation (2.3) are obtained for the same combination of P^* and T/λ_0 values as those used inequality (2.2) and are given in Table 2 for $\alpha = 2$.

Here the tables are developed for the sampling plans when the shape parameter $\alpha = 2$. The tables can be constructed for any other values of shape parameter α. If α is unknown, the general theory of maximum likelihood estimation or any other estimation procedures can be adopted to get an iterative solution for the shape parameter from the sample data and the estimated α is to be used to get α dependent tables.

Table 1: Minimum Sample Size for the specified ratio T/λ_0, confidence level P^*, acceptance number c, α=2 using binomial approximation

P^*	c	T/λ_0 2.75	3	3.25	3.5	3.75	4	4.25	4.5	4.75
0.75	0	1	1	1	1	1	1	1	1	1
0.75	1	2	2	2	2	2	2	2	2	2
0.75	2	4	4	3	3	3	3	3	3	3
0.75	3	5	5	5	4	4	4	4	4	4
0.75	4	6	6	6	6	5	5	5	5	5
0.75	5	7	7	7	7	6	6	6	6	6
0.75	6	9	8	8	8	8	7	7	7	7
0.75	7	10	9	9	9	9	9	8	8	8
0.75	8	11	11	10	10	10	10	9	9	9
0.75	9	12	12	11	11	11	11	10	10	10
0.75	10	13	13	12	12	12	12	12	11	11
0.9	0	2	1	1	1	1	1	1	1	1
0.9	1	3	3	3	3	2	2	2	2	2
0.9	2	4	4	4	4	4	4	3	3	3
0.9	3	6	5	5	5	5	5	5	4	4
0.9	4	7	7	6	6	6	6	6	6	5
0.9	5	8	8	7	7	7	7	7	7	6
0.9	6	9	9	9	8	8	8	8	8	8
0.9	7	11	10	10	9	9	9	9	9	9
0.9	8	12	11	11	11	10	10	10	10	10
0.9	9	13	13	12	12	11	11	11	11	11
0.9	10	14	14	13	13	13	12	12	12	12
0.95	0	2	2	2	2	1	1	1	1	1
0.95	1	3	3	3	3	3	3	3	2	2
0.95	2	5	4	4	4	4	4	4	4	4
0.95	3	6	6	5	5	5	5	5	5	5
0.95	4	7	7	7	6	6	6	6	6	6
0.95	5	9	8	8	8	7	7	7	7	7
0.95	6	10	9	9	9	9	8	8	8	8
0.95	7	11	11	10	10	10	9	9	9	9
0.95	8	13	12	11	11	11	10	10	10	10
0.95	9	14	13	13	12	12	12	11	11	11
0.95	10	15	14	14	13	13	13	12	12	12
0.99	0	3	2	2	2	2	2	2	2	2
0.99	1	4	4	4	4	3	3	3	3	3
0.99	2	6	5	5	5	5	4	4	4	4
0.99	3	7	7	6	6	6	6	5	5	5
0.99	4	9	8	8	7	7	7	7	6	6
0.99	5	10	9	9	8	8	8	8	7	7
0.99	6	11	11	10	10	9	9	9	9	8
0.99	7	13	12	11	11	10	10	10	10	9
0.99	8	14	13	13	12	12	11	11	11	11
0.99	9	15	14	14	13	13	12	12	12	12
0.99	10	16	16	15	14	14	14	13	13	13

Table 2: Minimum Sample Size for the specified ratio $T/\lambda_{0,}$ confidence level P^*, acceptance number c, α=2 using Poisson approximation

P^*	c	T/λ_0								
		2.75	3	3.25	3.5	3.75	4	4.25	4.5	4.75
0.75	0	3	3	3	3	3	3	3	3	3
0.75	1	5	4	4	4	4	4	4	4	4
0.75	2	6	6	6	6	6	6	6	6	5
0.75	3	7	7	7	7	7	7	7	7	7
0.75	4	9	8	8	8	8	8	8	8	8
0.75	5	10	10	10	9	9	9	9	9	9
0.75	6	11	11	11	11	10	10	10	10	10
0.75	7	13	12	12	12	12	12	11	11	11
0.75	8	14	13	13	13	13	13	13	13	12
0.75	9	15	15	14	14	14	14	14	14	14
0.75	10	16	16	16	15	15	15	15	15	15
0.9	0	4	4	4	4	4	4	4	4	4
0.9	1	6	6	6	6	6	6	6	5	5
0.9	2	8	7	7	7	7	7	7	7	7
0.9	3	9	9	9	9	9	8	8	8	8
0.9	4	11	10	10	10	10	10	10	10	10
0.9	5	12	12	12	11	11	11	11	11	11
0.9	6	14	13	13	13	13	12	12	12	12
0.9	7	15	15	14	14	14	14	14	14	13
0.9	8	16	16	16	15	15	15	15	15	15
0.9	9	18	17	17	17	16	16	16	16	16
0.9	10	19	19	18	18	18	17	17	17	17
0.95	0	5	5	5	5	5	5	5	5	5
0.95	1	7	7	7	7	6	6	6	6	6
0.95	2	9	8	8	8	8	8	8	8	8
0.95	3	10	10	10	10	10	10	9	9	9
0.95	4	12	12	11	11	11	11	11	11	11
0.95	5	13	13	13	13	13	12	12	12	12
0.95	6	15	15	14	14	14	14	14	14	14
0.95	7	17	16	16	15	15	15	15	15	15
0.95	8	18	17	17	17	17	16	16	16	16
0.95	9	19	19	18	18	18	18	18	18	17
0.95	10	21	20	20	20	19	19	19	19	19
0.99	0	7	7	6	6	6	6	6	6	6
0.99	1	9	9	9	9	8	8	8	8	8
0.99	2	11	11	11	10	10	10	10	10	10
0.99	3	13	13	12	12	12	12	12	12	12
0.99	4	15	14	14	14	14	14	13	13	13
0.99	5	16	16	16	15	15	15	15	15	15
0.99	6	18	18	17	17	17	17	16	16	16
0.99	7	20	19	19	19	18	18	18	18	18
0.99	8	21	21	20	20	20	20	19	19	19
0.99	9	23	22	22	21	21	21	21	21	21
0.99	10	24	24	23	23	23	22	22	22	22

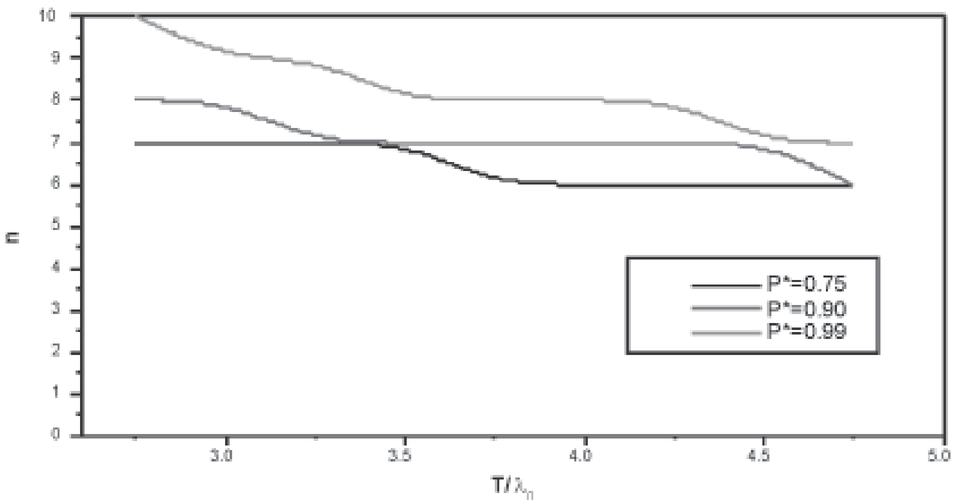

Figure 1: The plot for the minimum sample size versus T/λ_0 with α=2 and c=5

The probability of acceptance can be regarded as a function of the deviation of specified average from the true average. This function is called operating characteristic (OC) function for the sampling plan. Thus the operating characteristic (OC) function for the sampling plan (n, c, T/λ_0) gives the probability of accepting a lot and it is given as

$$L(p) \quad = \quad \sum_{i=0}^{c} \binom{n}{i} p^i \; (1-p)^{n-i} \tag{2.4}$$

where $p = F(T/\lambda)$ is considered as a function of the lot quality parameter λ. It can be seen that the operating characteristic is an increasing function λ. For a given, P*, T/λ_0, the choice of c and n will be made on the basis of operating characteristics. For some sampling plan, the values of the operating characteristic as a function of λ/λ_0 are displayed in Table 3.

The producer's risk is the probability of rejecting a good lot. For a specified value of the producer risk say 0.05, one may be interested in knowing what value of λ or λ/λ_0 will ensure the producer's risk less than or equal to 0.05 for a given

Table 3: Values of the Operating Characteristic function of the sampling plan (n, c, T/λ_0) for given confidence level $P^*$with α =2

P^*	n	c	T/λ_0	T/λ_0					
				2	4	6	8	10	12
0.75	4	2	2.75	0.5955	0.9508	0.9911	0.9977	0.9993	0.9997
0.75	4	2	3.00	0.5187	0.9317	0.9869	0.9965	0.9988	0.9995
0.75	3	2	3.25	0.7317	0.9704	0.9946	0.9986	0.9995	0.9998
0.75	3	2	3.50	0.6819	0.9607	0.9925	0.9980	0.9993	0.9997
0.75	3	2	3.75	0.6317	0.9493	0.9899	0.9973	0.9991	0.9996
0.75	3	2	4.00	0.5821	0.9362	0.9867	0.9963	0.9987	0.9995
0.75	3	2	4.25	0.5338	0.9215	0.9829	0.9951	0.9983	0.9993
0.75	3	2	4.50	0.4874	0.9051	0.9784	0.9937	0.9977	0.9991
0.75	3	2	4.75	0.4433	0.8873	0.9732	0.9919	0.9971	0.9988
0.9	4	2	2.75	0.5955	0.9508	0.9911	0.9977	0.9993	0.9997
0.9	4	2	3.00	0.5187	0.9317	0.9869	0.9965	0.9988	0.9995
0.9	4	2	3.25	0.4460	0.9090	0.9814	0.9949	0.9983	0.9993
0.9	4	2	3.50	0.3790	0.8828	0.9746	0.9928	0.9975	0.9990
0.9	4	2	3.75	0.3188	0.8535	0.9662	0.9902	0.9965	0.9986
0.9	4	2	4.00	0.2657	0.8213	0.9563	0.9869	0.9953	0.9980
0.9	3	2	4.25	0.5338	0.9215	0.9829	0.9951	0.9983	0.9993
0.9	3	2	4.50	0.4874	0.9051	0.9784	0.9937	0.9977	0.9991
0.9	3	2	4.75	0.4433	0.8873	0.9732	0.9919	0.9971	0.9988
0.95	5	2	2.75	0.3918	0.8994	0.9800	0.9947	0.9982	0.9993
0.95	4	2	3.00	0.5187	0.9317	0.9869	0.9965	0.9988	0.9995
0.95	4	2	3.25	0.4460	0.9090	0.9814	0.9949	0.9983	0.9993
0.95	4	2	3.50	0.3790	0.8828	0.9746	0.9928	0.9975	0.9990
0.95	4	2	3.75	0.3188	0.8535	0.9662	0.9902	0.9965	0.9986
0.95	4	2	4.00	0.2657	0.8213	0.9563	0.9869	0.9953	0.9980
0.95	4	2	4.25	0.2197	0.7867	0.9448	0.9829	0.9937	0.9974
0.95	4	2	4.50	0.1803	0.7503	0.9317	0.9782	0.9918	0.9965
0.95	4	2	4.75	0.1471	0.7125	0.9170	0.9726	0.9896	0.9955
0.99	6	2	2.75	0.2418	0.8350	0.9641	0.9900	0.9966	0.9987
0.99	5	2	3.00	0.3114	0.8643	0.9710	0.9920	0.9973	0.9989
0.99	5	2	3.25	0.2431	0.8242	0.9595	0.9884	0.9959	0.9984
0.99	5	2	3.50	0.1868	0.7801	0.9456	0.9837	0.9942	0.9976
0.99	5	2	3.75	0.1416	0.7328	0.9291	0.9780	0.9920	0.9967
0.99	4	2	4.00	0.2657	0.8213	0.9563	0.9869	0.9953	0.9980
0.99	4	2	4.25	0.2197	0.7867	0.9448	0.9829	0.9937	0.9974
0.99	4	2	4.50	0.1803	0.7503	0.9317	0.9782	0.9918	0.9965
0.99	4	2	4.75	0.1471	0.7125	0.9170	0.9726	0.9896	0.9955

sampling plan. The value of λ and hence the value of λ/λ_0 is the smallest positive number so that $p = F(T/\lambda) = F\left[\left(\frac{T}{\lambda_0}\frac{\lambda_0}{\lambda}\right)\right]$ satisfies the following inequality

$$\sum_{i=0}^{c}\binom{n}{i} p^i (1-p)^{n-i} \geq 0.95$$

For a given sampling plan $(n, c, T/\lambda_0)$ at a specified confidence level P^* (i.e., consumer's risk$_{(1-P^*)}$), the smallest values of λ/λ_0 satisfying the inequality (2.5) are displayed in Table 4. In this paper, the tables have been constructed only for α=2 owing to the problem of space and the computer programs to produce tables for the other shape parameters are available with the authors.

Figure 1 is the plot of the required minimum sample size versus T/λ_0 for some selected values of the confidence level. It shows that the required minimum sample size decreases as the values of T/λ_0 increases and the required minimum sample sizes get close for all specified confidence levels when the value of T/λ_0 is large. Figure 2 is the plot of the operating characteristic as a function of T/λ_0 for some

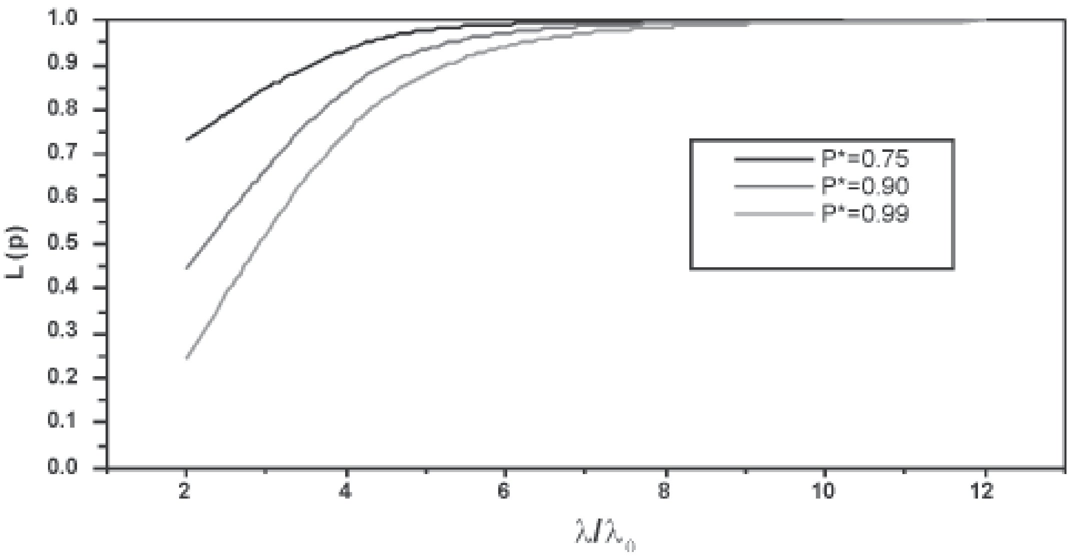

Figure 2: The plot for the Operating chracteristic as a function of λ/λ_0 for T/λ_0=3.25 , α =2 and c=2

selected values of the confidence level and it can be seen that the operating characteristic is an increasing function of λ for all selected values of the confidence levels.

3. DESCRIPTIONS OF THE TABLES AND EXAMPLES

Assume that the lifetime distribution of the product under study follows a GE distribution with $\alpha = 2$ and that the experimenter wants to know the minimum sample size to be considered to make a decision (accepting or rejecting the lot), when one wants the true average life should be at least 1000 hours with the probability of accepting a bad lot less than or equal to 0.25 or $P^* = 0.75$. The experimenter wish to stop the experiment at T=2750 hours. Then for an acceptance number c=2, the required sample size corresponding to the values of $P^* = 0.75$, $_{T/\lambda_0}$ =2.75 and c=2 is n=4 provided in Table 1. Thus 4 units have to be put on test. If during 2750 hours, no more than 2 failures out of 4 are observed, then the lot can be accepted with the assurance that the true average life is at least 1000 hours with a confidence level of $P^* = 0.75$. If the Poisson approximation to binomial probability is used, the value of n=6 is obtained for the same combination c, T/λ_0, P^* from Table 2.

For the sampling plan $(n, c, T/\lambda_0)$ $(n = 4, c = 2, T/\lambda_0 = 2.75)$, P^* 0.75 under GE model with α=2, the operating characteristic values from Table 3 are

λ/λ_0	2	4	6	8	10	12
L(p)	0.5955	0.9508	0.9911	0.9977	0.9993	0.9997

This shows that, if the true average life is twice the specified average life (i.e, $_{\lambda/\lambda_0}$= 2), the producer's risk is approximately 0.4045.

From Table 4, we can get the smallest values of λ/λ_0 for various choices of c, T/λ_0 in order to assert the producer's risk is less than or equal to 0.05. Thus, in the above example, the smallest value of λ/λ_0 is 3.98 for c=2, T/λ_0=2.75 and P^*=0.75. That is, the products should have a average lifetime of at least 3.98 times of the specified average lifetime of 1000 hours in order that the lot will be accepted with probability 0.95 under the design. The actual average lifetime necessary to transship 95 percent of the lots is provided in Table 4.

Alternatively, one can get the sampling plans as follows: Assume that the lifetimes of products under test follow an GE distribution with α=2, and consumers

Table 4: Minimum ratio of true λ and required λ_0 for the acceptability of a lot with producer's risk of 0.05 for $\alpha = 2$

P^*		T/λ_0								
	c	2.75	3	3.25	3.5	3.75	4	4.25	4.5	4.75
0.75	0	10.87	11.85	12.84	13.83	14.82	15.81	16.79	17.78	18.77
0.75	1	4.30	4.69	5.08	5.47	5.86	6.25	6.64	7.03	7.42
0.75	2	3.98	4.35	3.48	3.75	4.02	4.28	4.55	4.82	5.09
0.75	3	3.13	3.41	3.69	3.01	3.22	3.44	3.65	3.87	4.08
0.75	4	2.64	2.88	3.12	3.37	2.78	2.96	3.15	3.33	3.52
0.75	5	2.33	2.55	2.76	2.97	2.48	2.65	2.82	2.98	3.15
0.75	6	2.47	2.31	2.50	2.69	2.88	2.43	2.58	2.73	2.89
0.75	7	2.27	2.13	2.31	2.49	2.66	2.84	2.41	2.55	2.69
0.75	8	2.11	2.31	2.16	2.32	2.49	2.66	2.27	2.40	2.53
0.75	9	1.99	2.17	2.04	2.19	2.35	2.51	2.16	2.28	2.41
0.75	10	1.88	2.05	1.94	2.09	2.24	2.39	2.53	2.18	2.31
0.9	0	15.87	11.85	12.84	13.83	14.82	15.81	16.79	17.78	18.77
0.9	1	6.00	6.54	7.09	7.63	5.86	6.25	6.64	7.03	7.42
0.9	2	3.98	4.35	4.71	5.07	5.43	5.80	4.55	4.82	5.09
0.9	3	3.74	3.41	3.69	3.98	4.26	4.55	4.83	5.11	5.40
0.9	4	3.13	3.42	3.12	3.37	3.61	3.85	4.09	4.33	3.52
0.9	5	2.75	3.00	2.76	2.97	3.18	3.39	3.61	3.82	3.15
0.9	6	2.47	2.70	2.92	2.69	2.88	3.08	3.27	3.46	3.65
0.9	7	2.55	2.48	2.68	2.49	2.66	2.84	3.02	3.20	3.37
0.9	8	2.37	2.31	2.50	2.69	2.49	2.66	2.82	2.99	3.15
0.9	9	2.22	2.42	2.35	2.53	2.35	2.51	2.66	2.82	2.98
0.9	10	2.10	2.29	2.23	2.40	2.57	2.39	2.53	2.68	2.83
0.95	0	15.87	17.31	18.75	20.20	14.82	15.81	16.79	17.78	18.77
0.95	1	6.00	6.54	7.09	7.63	8.18	8.72	9.27	7.03	7.42
0.95	2	4.82	4.35	4.71	5.07	5.43	5.80	6.16	6.52	6.88
0.95	3	3.74	4.08	3.69	3.98	4.26	4.55	4.83	5.11	5.40
0.95	4	3.13	3.42	3.70	3.37	3.61	3.85	4.09	4.33	4.57
0.95	5	3.11	3.00	3.25	3.50	3.18	3.39	3.61	3.82	4.03
0.95	6	2.79	2.70	2.92	3.15	3.37	3.08	3.27	3.46	3.65
0.95	7	2.55	2.78	2.68	2.89	3.10	2.84	3.02	3.20	3.37
0.95	8	2.59	2.58	2.50	2.69	2.88	2.66	2.82	2.99	3.15
0.95	9	2.43	2.42	2.62	2.53	2.71	2.89	2.66	2.82	2.98
0.95	10	2.29	2.29	2.48	2.40	2.57	2.74	2.53	2.68	2.83
0.99	0	19.72	17.31	18.75	20.20	21.64	23.08	24.52	26.32	29.01
0.99	1	7.34	8.01	8.68	9.34	8.18	8.72	9.27	9.81	10.36
0.99	2	5.54	5.26	5.69	6.13	6.57	5.80	6.16	6.52	6.88
0.99	3	4.27	4.66	4.42	4.76	5.10	5.44	4.83	5.11	5.40
0.99	4	3.95	3.89	4.21	3.99	4.27	4.56	4.84	4.33	4.57
0.99	5	3.44	3.39	3.67	3.50	3.75	3.99	4.24	3.82	4.03
0.99	6	3.07	3.35	3.29	3.55	3.37	3.60	3.82	4.05	3.65
0.99	7	3.04	3.06	3.01	3.25	3.10	3.30	3.51	3.72	3.37
0.99	8	2.81	2.83	3.07	3.01	3.44	3.07	3.27	3.46	3.65
0.99	9	2.62	2.65	2.87	2.82	3.02	2.89	3.07	3.25	3.43
0.99	10	3.21	3.50	2.70	2.67	2.86	3.05	2.91	3.08	3.25

require a probability of rejecting a bad lot, P^*=0.75, and the sampling plan is based on an acceptance number c=2 and T/λ_0=2.75. What should the true average lifetime of products fulfill so that the producers risk will be 0.05? From Table 4 one can get the smallest value of λ/λ_0 is 3.98. Thus, the products under test should have an average lifetime at least 6.89 times of the specified one in order that the product can be accepted with probability 0.95 under the sampling design. From Table 1 we can find that the number of products required to be tested is n=4 and the sampling plan is $(n,\ c,\ T/\lambda_0) = (4,\ 2,\ 2.75)$.

Numerical Example: Consider a problem associated with the failure times of the air conditioning system of an air plane provided by Bain and Engelhardt (1991, page 101). This data can be regarded as an ordered sample of size 11 with observations (x_i, i = 1, ..., 11): 33, 47, 55, 56, 104, 176, 182, 220, 239, 246, 320

First the generalized exponential distribution is fitted to the given data. The MLEs of α and λ are 2.2355 and 0.0104 respectively. The Kolmogorov-Smirnov distance between the observed and fitted distribution function is 0.2194 with the associated p value 0.5978. Therefore, generalized exponential distribution fit quite well to the given data

Let the specified average life be 50 hours and the testing time be 50 hours, this leads to ratio T/λ_0 =1.00 with corresponding n and c as 11, 2 from Table 5 for P^*=0.75. Therefore, the sampling plan for the above sample data is (n = 11, c = 2, T/λ_0 = 1.00). Based on these observations, an experimenter can decide whether to accept or reject the lot. The experimenter accept the lot only if the number of failures before 50 hours is less than or equal to 2. In the given sample only two failures are recorded before T=50 hours at the time instances 33 and 47. Therefore the experimenter can accept the lot with the assurance that the average failure time is at least 50 hours with a confidence level of $P^* = 0.75$.

Table 5: Minimum Sample Size for the specified ratio T/λ_0, confidence level P^*, acceptance number c, $\alpha = 2.2355$ using binomial approximation

	P*=0.75	P*=0.90	P*=0.95	P*=0.99
(n, c, T/λ_0)	(11,2,1.0)	(14,2,1.0)	(16,2,1.0)	(21,2,1.0)

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